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Sharp bounds for complier average potential outcomes in experiments with noncompliance and incomplete reporting

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ABSTRACT

Published reports of experiments with noncompliance often fail to report information necessary for recovering average potential outcomes for compliers. We derive sharp bounds on the average potential outcomes for compliers, when given only average outcomes for units assigned to treatment, average outcomes for units assigned to control, and the difference in average take-up between assignment tuses.

1. Introduction

When reporting results from experiments with noncompliance, including encouragement designs, researchers often include estimates of the average outcomes in treatment, average outcomes in control and the difference in average take-up between assignment tuses. These figures are sufficient for estimating a complier average causal effect (CACE), sometimes referred to as a local average treatment effect (Angrist et al., 1996). Although sufficient information is typically estimable from the data (Imbens and Rubin, 1997), these three figures alone do not provide enough information to identify the average potential outcomes for compliers. Subntive interpretation of average causal effects often requires knowledge of baseline outcomes. For example, an increase of 10 points in test scores may be crucial if the baseline average score is 60 out of 100, but of little consequence when the baseline average score is 10 out of 100. Since compliers are the referent population for causal effects, it is their baseline outcomes that are of interest.

We have noted that many published studies do not provide enough information about the joint distribution of outcomes, treatment, and assignment for backing out complier average potential outcomes. To assess the magnitude of this problem, we collected every published study (to our knowledge) reporting randomized experiments on get-out-the-vote efforts using canvassing and found that only 6 of the 20 studies reported enough information to recover complier average potential outcomes.[1] In this note, we provide a method for partial identification of complier potential outcomes when only limited information is available in the reported results. Applying the method to an encouragement design, we show that the bounds produced by this method, although sharp, may be subntively uninformative. Given that little additional information is required to yield point identification, we conclude with simple recommendations for reporting practice.

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[1] The list of studies and coding is available at http://pantheon.yale.edu/~pma5/meta.csv.

2. Setting

We follow the setup of Angrist and Pischke (2009, Chapter 4). Let $Y_i(d, z)$ be the potential outcome of unit i under treatment d

$\in$ {0, 1} and assignment $z \in$ {0, 1}, let $D_i(z)$ be the potential treatment of unit i under assignment z, and let Z_i be the treatment assigned to unit i. The causal effect for unit i is then $Y_i(1, Z_i) — Y_i(0, Z_i)$. We assume (i) random assignment: [Y_i(d, z), $D_i(z)$: d, $z \in$ {0, 1}] $\perp\!\!\!\perp$ Z_i; (ii) exclusion restriction: $Y_i(d, z)$ does not depend on z, so we may write Y_i(d, z) = Y_i(d); (iii) non-zero first-ge effect: E [$D_i(1) — D_i(0)$] = 0; (iv) monotonicity: $\forall i$, $D_i(1) > D_i(0)$. We make two notational simplifications. First, given (i), labels are exchangeable and so we drop notational dependence on i. Second, we define $P_c \equiv$ Pr[D(1) > D(0)], $EY_1 \equiv$ E [Y|Z $\equiv$ 1], and EY_0 = E [Y|Z = 0].

This setting implies that the "LATE theorem" (Angrist and Pischke, 2009, Theorem 4.4.1) holds: the CACE, E [Y(1) — Y(0)|D(1) > D(0)], is equal to

$$\frac{\mathrm{E}[Z=1]-\mathrm{E}[Y|Z=0]}{\mathrm{E}[Z=1]-\mathrm{E}[D|Z=0]}=\frac{\mathrm{EY}_1-\mathrm{EY}_0}{\mathrm{P_c}}$$

where each component may be estimated using sample analogues to population quantities. The associated estimator is logically equivalent to the Wald instrumental variables estimator (Angrist et al., 1996). We make one additional assumption to ensure partial identification: we require that outcomes are bounded. Without loss of generality, fix $Y \in$ [0, 1].

3. Results

Proposition 1 asserts the intervals for complier potential outcomes identified under incomplete reporting.

Proposition 1. *When only* EY_1, EY_0 *and* P_c *are known, sharp bounds exist for the complier average potential outcomes. The average control potential outcome for compliers,* E [Y (0)|D(1) > D(0)] *lies in*

$$\left[\max\left\{0,-\frac{\mathrm{EY}_1-\mathrm{EY}_0}{\mathrm{P_c}},1-\frac{1-\mathrm{EY}_0}{\mathrm{P_c}}\right\},\min\left\{1,1-\frac{\mathrm{EY}_1-\mathrm{EY}_0}{\mathrm{P_c}},\frac{\mathrm{EY}_0}{\mathrm{P_c}}\right\}\right].$$

Similarly, the average treated potential outcome for compliers, E[Y(1)|D(1) > D(0)], lies in

$$\left[\max\left\{0,-\frac{\mathrm{EY}_1-\mathrm{EY}_0}{\mathrm{P_c}},1-\frac{1-\mathrm{EY}_1}{\mathrm{P_c}}\right\},\min\left\{1,1+\frac{\mathrm{EY}_1-\mathrm{EY}_0}{\mathrm{P_c}},\frac{\mathrm{EY}_1}{\mathrm{P_c}}\right\}\right].$$

Proof. Define EY_n = E [Y |D(1) = D(0)], EY_c (0) $\equiv$ E [Y(0)|D(1) > D(0)] and $EY_c(1) \equiv$ E [Y(1)|D(1) > D(0)]. By the law of total probability,

$P_cEY_c(0) + (1 — P_c)EY_n = EY_0$,

$P_cEY_c(1) + (1 — P_c)EY_n = EY_1$,

given the constraints that $0 \leq EY_c(0) \leq 1$, $0 \leq EY_c(1) \leq 1$, and $0 \leq EY_n \leq 1$. Since $EY_n = [EY_1 — P_cEY_c(1)]/(1 — P_c)$, we may reframe the problem as

$P_cEY_c(0) + EY1 — P_cEY_c(1) = EY_0$,

such that $0 < EY_c(0) < 1$, $0 < EY_c(1) \leq 1$,and$0 \leq EY_1 — P_cEY_c(1) \leq 1$. Then substituting the implied value of EY_c (1) from Eq. (3) into the system of inequalities and rearranging terms, we obtain

$0 \leq EY_c(1) \leq 1$,

$(EY_1 — EY_0)/P_c < EY_c(1) \leq 1 + (EY_1 — EY_0)/P_c$,

$(EY_1 + P_c — 1)/P_c \leq EY_c(1) \leq EY_1/P_c$,

which yields the interval for EY_c (1) asserted by the proposition. The interval for EY_c (0) follows via substitution from the equality in Eq. (3). □

The bounds will tend to be small when (i) the complier average causal effect is close to —1 or 1, (ii) average potential outcomes are close to 0 or 1, or (iii) the proportion of compliers is close to 1.

For contrast, we te sufficient conditions for point identification in Corollary 1, following the well-known result in, e.g., Imbens and Rubin (1997).

Corollary 1. *If E [Y |D = d, Z = z] and Pr[D = d|Z = z],for d, z e {0, 1}, are known, the complier average potential outcomes are point identified. E [Y(0) |D(1) > D(0)] is equal to*

$$\frac{\mathrm{EY}_0-\Pr[Z=0]\,\mathrm{E}[D=1,Z=0]-\Pr[Z=1]\,\mathrm{E}[D=0,Z=1]}{\mathrm{P_c}}$$

and E[Y(1)|D(1) > D(0)] = E[Y(0)|D(1) > D(0)] + $(EY_1 – EY_0)/P_c$.

Proof. The average outcome for non-compliers, E [Y|D(1) = D(0)], is equal to

$$\frac{\Pr[Z=0]\,\mathrm{E}[D=1,Z=0]+\Pr[Z=1]\,\mathrm{E}[D=0,Z=1]}{\Pr[Z=0]+\Pr[Z=1]}.$$

Substitution of E [Y|D(1) = D(0)] into Eqs. (1) and (2) yields the result. □

4. Application

We consider the encouragement study reported by Mullainathan et al. (2010), which randomly assigned an encouragement to watch a mayoral debate (Z) in order to assess the effect of watching (D) on changed views about at least one candidate (Y). The experiment had two-sided noncompliance, as both Pr[D = 0|Z = 1] > 0 and Pr[D = 1 |Z = 0] > 0. The authors report estimates of $EY_0 = 0.45$, $EY_1 = 0.51$ and $P_c = 0.21$. While the CACE can be estimated as 29 percentage points, the authors do not provide sufficient information for point estimation of complier average potential outcomes. We now estimate sharp bounds on complier potential outcomes by application of Proposition 1. The bounds are rather wide: E [Y(0)|D(1) >D(0))] ∈ [0, 0.71] and E [Y(1)|D(1) > D(0)] ∈ [0.29, 1]. Thus, although the CACE is identified, very little information is conveyed on complier potential outcomes by the published results.

Point identification is possible when additional information about the joint distribution of *Y*, D, *Z* is known. Since the replication data set is publicly available for the study[2], we are able to fill in the necessary missing information: Pr[*D* = 1|Z = 0] = 0.163, Pr[D = 0|Z = 1] = 0.634, E [Y|D = 1, *Z* = 0] = 0.506 and E [Y|D = 0, *Z* = 1] = 0.442. Application of Corollary 1 yields E [Y(0)|D(1) > D(0)] = 0.43 and E [Y(1)|D(1) > D(0)] = 0.72. Our inferences are considerably tightened by knowledge of additional features of the distribution that were not reported.

5. Discussion

Although estimation of the CACE relies only on EY_0, EY_1 and P_c, relevant information about compliers is conveyed by the joint distribution of *Y*, *D* and *Z*. Our results indicate that researchers should report estimates of E [Y|D = d, *Z* = *z*] and Pr[D = d|Z = z], for d, $z \in \{0, 1\}$, so that complier average potential outcomes can be estimated from the published results. Such reporting practices would have the additional benefit of facilitating alternative parametric models (e.g., bivariate probit) or robust variance estimators in the case where *Y* is binary.

References

Angrist, J.D., Imbens, G.W., Rubin, D.B., 1996. Identification of causal effects using instrumental variables. Journal of the American tistical Association 91, 444-455.

Angrist, J.D., Pischke, J.S., 2009. Mostly Harmless Econometrics: An Empiricist's Companion. Princeton University Press, Princeton, NJ.

Imbens, G.W., Rubin, D.B., 1997. Estimating outcome distributions for compliers in instrumental variables models. Review of Economic Studies 4,555-574.

Mullainathan, S., Washington, E., Azari, J.R., 2010. The impact of electoral debate on public opinions: an experimental investigation of the 2005 New York City mayoral election. In: Shapiro, I., Stokes, S., Wood, E., Kirshner, A.S. (Eds.), Political Representation. Cambridge University Press.

[2] The replication data set for Mullainathan et al. (2010) is archived at http://isps.research.yale.edu/data/D049/.